Computer Algebra Systems – CAS
A Technology that is Revolutionizing Mathematics Teaching!

Published in: is – International School Magazine, Vol. 8, Issue 3, Summer 2004

We are presently in an era that historians will certainly refer to in years to come as the Age of Technology. Similar to the Industrial Age, the technology revolution is making its presence felt in all areas of human endeavours, even those that were previously considered bastions of resistance to its seduction, e.g., the creative arts. The world of education is certainly no exception, and the single most important concern that all educators share globally regardless of level of instruction or subject matter is the appropriate use of technology in classroom curriculum and assessment.

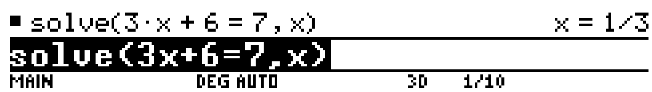
The recent evolution in hand-held technology, from four-function calculators to scientific calculators, then graphical display calculators and now calculators with the ability to perform symbolic computation has provoked fierce emotions from an otherwise traditionally cool-headed group of educators, namely mathematicians. Reactions toward this latest development range from outrage at what is seen as the beginning of the end of all mathematical "by hand" computation, hence mathematical understanding, to euphoria at the new possibilities this technology provides for increased accessibility of this traditionally elitist subject. Regardless of where in the spectrum of emotion an individual mathematics teacher may find herself, the fact remains that this technology is here to stay. At most we can succeed with great collective effort and energy in reducing the speed of its takeover, but it is a historically proven fact that it is impossible to stem the tide of technological advancement.

At the Vienna International School we decided to focus our energies, not in fighting technological advancement, but in aiding its evolution to the benefit of our students and teachers, and hopefully to our profession. For the past seven years I have been using computer algebra systems in all my classes, grade 6 through 12, including IB classes. (The IB does not currently permit the use of computer algebra technology in its external assessment papers.) We work closely with our host community in sharing our experiences and in engaging in philosophical debates over the appropriate use of this technology. Austria is the first country to have government supported research projects to assess the effects of the introduction of this technology into the mathematics curriculum. We are fortunate indeed to work with a dedicated and competent group of mathematics educators here, and it has been a credit to our school management that they continue to possess the vision and confidence to provide both moral and financial support for our pioneering endeavours.

In my classes I continue to witness countless ways in which this technology is helping my students gain greater understanding in exactly the skills the calculator can do for them, i.e., symbolic computation. When introducing equation solving in algebra it is a common mistake for students to choose the wrong equivalence transformation in their attempt to solve the equation. For example, ask a beginning algebra student to solve

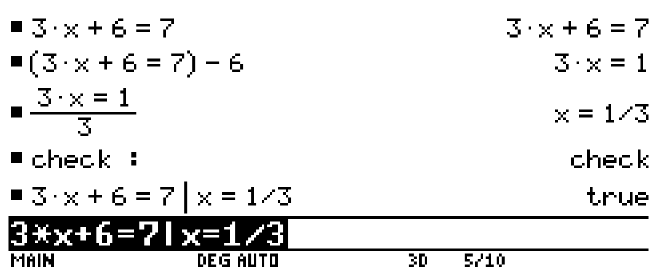
3x+6=7. A typical erroneous solution could look as follows: subtract 6 from both sides and arrive at 3x=1. Then divide by 1 and the result is 3. Or students could choose to subtract 3 at this stage and arrive at x=-2, or subtract 1 and arrive at x=2. In any case, the student is convinced she has performed this correctly and proceeds to the next problem. The lag time in obtaining feedback from the teacher inhibits optimal learning since the student may not be able to remember why she chose the equivalence transformation she did, and worse, not even particularly care at that stage.

Admittedly, there is not a lot to be learned about the process of equation solving if one uses the "solve" function of the CAS calculator to solve this equation. It would look like this:

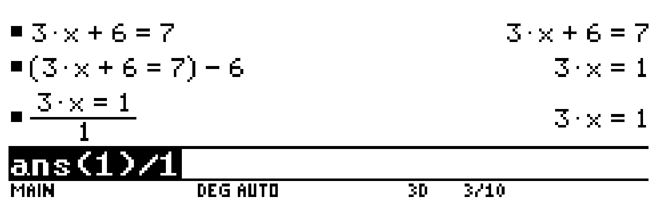


At most the pedagogical value of the "solve" function is that the student could attempt to find out where they went wrong by using the calculator as a check. At least the feedback is faster and helps the student remain motivated to discover possible errors.

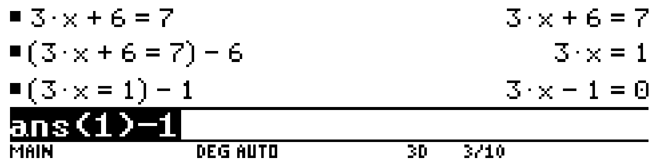
One of the strengths of this technology, however, is that it allows the student to perform the steps for solving the equation as she would manually. Assuming she chose the correct equivalence transformations, she would obtain the following:



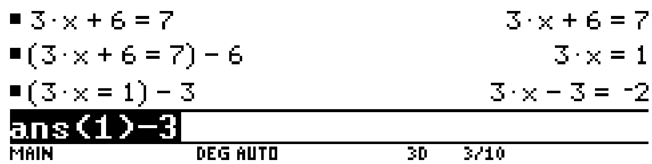
and be very pleased with herself! However, if she made one of the errors illustrated above, like divide by 1, she would see the following:



or if she subtracted 1:



or subtracted 3:



In all three cases where the incorrect equivalence transformation was chosen the student has received the instant feedback that the desired result was not obtained, i.e., the calculator did not show x= a number. The student chose the transformation in order to isolate the variable x, and this didn't happen! Psychologists say that if we receive a response to a stimulus within 10 seconds, it becomes "muscle memory". Indeed using the calculator as a trainer allows the student to verify their results immediately, thus optimizing understanding and retention of correct analytical manipulations. Once the student has understood the underlying mathematical concepts and procedures for solving equations, i.e., equivalence transformations, then in working with applications or mathematical modelling the teacher may allow the students to use the "solve" function since the emphasis in learning is no longer on the procedure for solving an equation, but rather in setting up the correct mathematical equation(s) to describe the real world situation, and then interpreting the solution the calculator renders in light of the given problem. The learning can focus on the higher order skills of mathematical modelling, i.e., translation and interpretation while allowing the calculator to perform the (sometimes tedious) lower-order algorithmic manipulation.

There are many other examples I could show in which the CAS calculators assist students to gain increased understanding of underlying algebraic structures, and hence improve analytical or algorithmic "by hand" computations. CAS however has many other capabilities that can contribute to the creation of a veritable mathematics laboratory in the classroom. Since it can generate many examples of a particular concept very quickly, students can experiment and arrive at conjectures as a result of the patterns they see. The calculator makes mathematical concepts accessible that may contain algorithms too difficult for them at a particular age, maturity or ability level. Even weak students can now glimpse beyond their borders into areas of mathematics previously denied them and experience the wonderful world of mathemagic!